No admittance under 4:

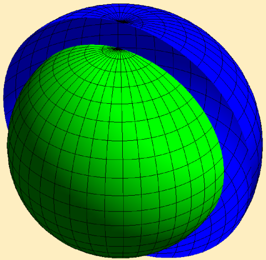
Four-fermion condensation in strongly interacting dense matter

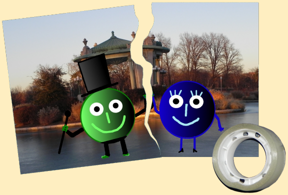
M. Alford, K. Schwenzer and A. Windisch

Der Wissenschaftsfonds.

Symposium Latsis EPFL (14-18 July 2014) on
Strong and Electroweak Matter (SEWM14)

Two species





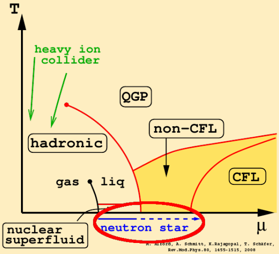
Contexts

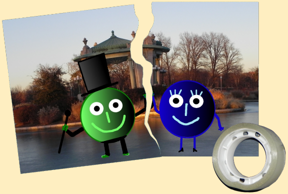
- ultracold atom systems
- solids
- quark matter
- neutron stars

Different Fermi momenta

- first constituent costs zero energy
- p, −p
- ⇒ second constituent costs free energy
- might not be compensated

QCD phase diagram





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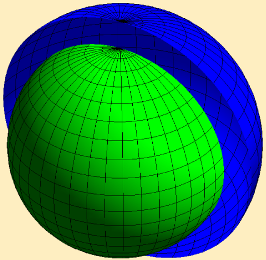
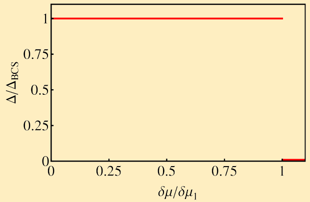




BCS vs. unpaired

- A. Clogston, Phys. Rev. Lett. 9, 266 (1962)
- B. Chandrasekhar, App. Phys. Lett 1, 7 (1962)

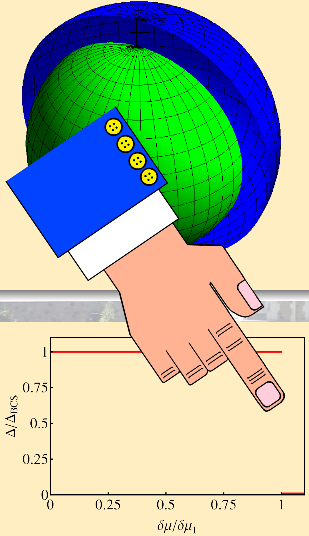
$$\mu_1 = \bar{\mu} + \delta\mu$$

$$\mu_2 = \bar{\mu} - \delta\mu$$

Chandrasekhar-Clogston Limit

- $\delta\mu < \delta\mu_1 = \frac{\Delta_0}{\sqrt{2}}$: BCS
- $\delta\mu = \delta\mu_1$: 1st order

Two species



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Chandrasekhar-Clogston Limit

- $\delta\mu < \delta\mu_1 = \frac{\Delta_0}{\sqrt{2}}$: **BCS**
- $\delta\mu = \delta\mu_1$: 1^{st} **order**

Abandoning homogeneity...

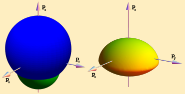
Inhomogeneous chiral condensation

Review by M. Buballa and S. Carignano,
***"Inhomogeneous chiral condensates"*,**
arXiv:1406.1367

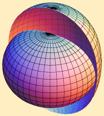
Inhomogeneous diquark condensation

Available on the market:

- **LOFF-condensation**



- **DFS-phase**



- A. Larkin and Y. Ovchinnikov, Zh. Eksp. Teor. Fiz 47, 1136 (1964)

  Translation: [Sov. Phys. JETP 20, 762 (1965)]
- P. Fulde and R. Ferrell, Phys. Rev. 135, A550 (1964)

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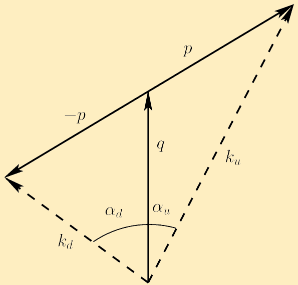
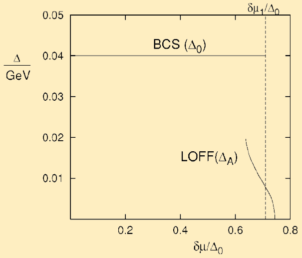
LOFF **FFLO**





- translational and rotational not invariant
- condensate varies as plane wave with 2q
- crystalline structure, $\Delta(r) = \cos(2\mathbf{q} \cdot \mathbf{r})$

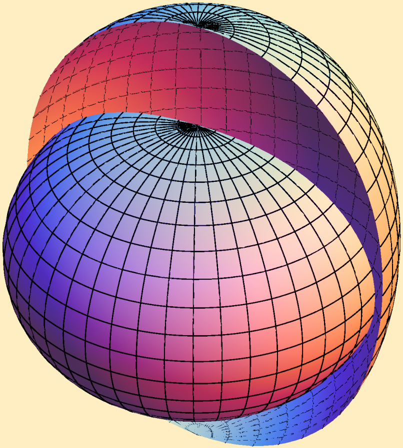
LOFF in QCD

M. Alford, J. Bowers and K. Rajagopal, Phys.Rev. D63 (2001)

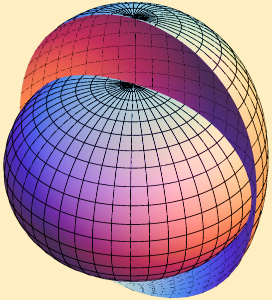
- crystalline condensate
- net momentum
- QM and glitches: vortex pinning

Deformed Fermi Surface – Phase

H. Müther and A. Sedrakian, PRL 88 (2002)



H. Müther and A. Sedrakian, PRL 88 (2002)



0^{th} and 2^{nd} polynomial

$$\mu_f = \mu_{f,0} + \mu_{f,2}\frac{1}{2}(3\cos^2\vartheta - 1)$$

Definition $\bar{\mu}$

$$\bar{\mu} = \mu_{f,0} - \frac{1}{2}\mu_{f,2}$$

Definition $\varepsilon_{S/A}$

$$\varepsilon_{S/A} = \frac{3}{4}\left(\frac{\mu_{2,d}}{\bar{\mu}_d} \pm \frac{\mu_{2,u}}{\bar{\mu}_u}\right)$$

Legendre Polynomials

$$\mu_f = \sum_{l=0}^{\infty} \mu_{f,l} P_l(\cos\vartheta)$$

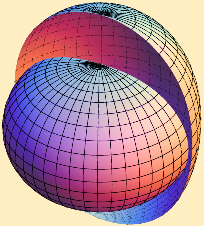
Actual deformation

$$\mu_f = \bar{\mu}_f(1 \pm \varepsilon_A \sin^2\vartheta)$$

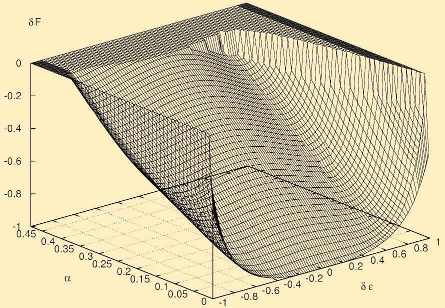
H. Müther and A. Sedrakian, PRL 88 (2002)

Superconducting vs. Normal State

H. Müther and A. Sedrakian, PRL 88 (2002)



Difference in free energy



H. Müther and A. Sedrakian, PRC 67 (2003)

Superconducting vs. Normal State (LOFF included)

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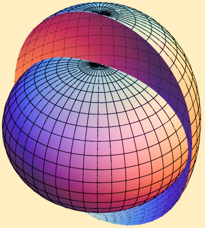

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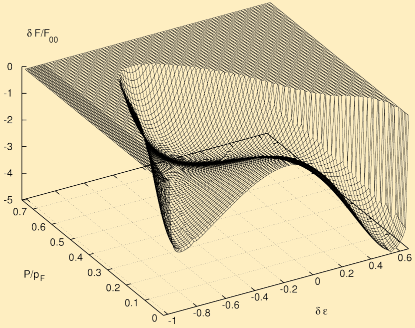

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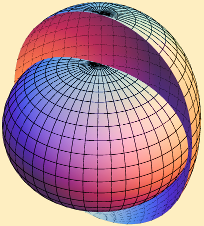
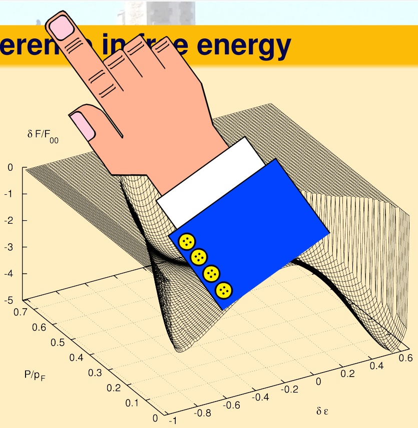
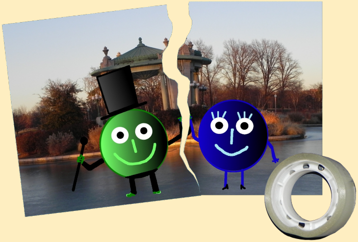


Difference in free energy



WHY QUARTETTING?

Pairing



Quartetting

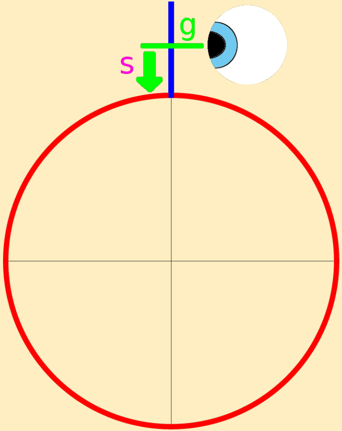
Kinematically suppressed

$$\langle qq \rangle = 0$$

Condensation?

$$\langle qqqq \rangle = ?$$

RG setting



Scaling behavior $\lim_{s\to 0}$

weak coupling, $\delta\mu \ll \Delta$

- $\langle qq \rangle$: **marginal**
- $\langle qqqq \rangle$: **irrelevant**

weak coupling, $\delta\mu \gtrsim \Delta$

- $\langle qq \rangle$: **irrelevant**
- $\langle qqqq \rangle$: **irrelevant**

strong coupling, $\delta\mu \gtrsim \Delta$

- $\langle qq \rangle$: **suppressed**
- $\langle qqqq \rangle$: **?**

Toy model: Bosonized Lagrangian $SU(2)_f \otimes SU(2)_c$

$$\mathscr{L} =$$

$$\bar{\psi}_A^{\alpha} \left(\partial\!\!\!/_{\mu} - (\mu + \delta\mu\sigma_3)\gamma^4 + m \right)_{AB}^{\alpha\beta} \psi_B^{\beta} + \frac{1}{2} \left(|\partial_\mu \Xi| \right)^2 + \frac{1}{2} \left(|\partial_\mu \Theta| \right)^2$$

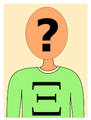
$$+ \frac{m_\Theta^2}{2} \Theta_{AB}^{\alpha\beta} \varepsilon_{ijkl} c_{\alpha A}^{i} c_{\beta B}^{j} c_{\gamma C}^{k} c_{\delta D}^{l} \Theta_{CD}^{\gamma\delta}$$

$$+ \frac{g_\Theta^Y}{2} \sqrt{\Xi^*} \, \varepsilon_{ijkl} c_{\alpha A}^{i} c_{\beta B}^{j} c_{\gamma C}^{k} c_{\delta D}^{l} \Theta_{AB}^{\alpha\beta} \psi_C^{\gamma} \psi_D^{\delta}$$

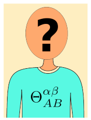
$$+ \frac{g_\Theta^Y}{2} \sqrt{\Xi} \, \varepsilon_{ijkl} c_{\alpha A}^{i} c_{\beta B}^{j} c_{\gamma C}^{k} c_{\delta D}^{l} \Theta_{AB}^{\alpha\beta} \bar{\psi}_C^{\gamma} \bar{\psi}_D^{\delta}$$

$$+ U(|\Xi|) + g_{AB}^{\alpha\beta} |\Xi| \Theta_{AC}^{\alpha\gamma} \Theta_{BC}^{\beta\gamma} + m_\Theta^2 \, \Theta_{AB}^{\alpha\beta} \Theta_{AB}^{\alpha\beta}$$

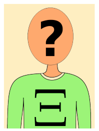
Player 1



Player 2



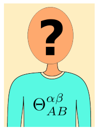
Player 1



Personal File

- ***Name:*** Ξ
- ***Species:*** **Boson**
- ***Occupation:*** **complex scalar**
- ***Represents:*** **4-fermion condensate**

Player 2



Personal File

- ***Name:*** $\Theta^{\alpha\beta}_{AB}$
- ***Species:*** **Ghost-like tensor field**
- ***Occupation:*** **complex, 0 baryon number**
- ***Represents:*** **2-fermion pairing**

Toy model: Bosonized Lagrangian $SU(2)_f \otimes SU(2)_c$

$$\mathscr{L} =$$

$$\bar{\psi}^{\alpha}_{A}\left(\partial\!\!\!/_{\mu} - (\mu + \delta\mu\sigma_3)\gamma^4 + m\right)^{\alpha\beta}_{AB}\psi^{\beta}_{B} + \frac{1}{2}\left(|\partial_\mu \Xi|\right)^2 + \frac{1}{2}\left(|\partial_\mu \Theta|\right)^2$$

$$+\frac{m^2_{\Theta}}{2}\Theta^{\alpha\beta}_{AB}\varepsilon_{ijkl}c^{i}_{\alpha A}c^{j}_{\beta B}c^{k}_{\gamma C}c^{l}_{\delta D}\Theta^{\gamma\delta}_{CD}$$

$$+\frac{g^{Y}_{\Theta}}{2}\sqrt{\Xi^*}\,\varepsilon_{ijkl}c^{i}_{\alpha A}c^{j}_{\beta B}c^{k}_{\gamma C}c^{l}_{\delta D}\Theta^{\alpha\beta}_{AB}\psi^{\gamma}_{C}\psi^{\delta}_{D}$$

$$+\frac{g^{Y}_{\Theta}}{2}\sqrt{\Xi}\,\varepsilon_{ijkl}c^{i}_{\alpha A}c^{j}_{\beta B}c^{k}_{\gamma C}c^{l}_{\delta D}\Theta^{\alpha\beta}_{AB}\bar{\psi}^{\gamma}_{C}\bar{\psi}^{\delta}_{D}$$

$$+U(|\Xi|) + g^{\alpha\beta}_{AB}|\Xi|\Theta^{\alpha\gamma}_{AC}\Theta^{\beta\gamma}_{BC} + m^2_{\Theta}\,\Theta^{\alpha\beta}_{AB}\Theta^{\alpha\beta}_{AB}$$

Flow equation

$$\frac{\partial}{\partial k}\Gamma_k = \frac{1}{2}\mathbf{Tr}\left\{\left[\Gamma^{(2)}_k + R_k\right]^{-1}\frac{\partial}{\partial k}R_k\right\}$$

Toy model: Bosonized Lagrangian $SU(2)_f \otimes SU(2)_c$

$$
\begin{aligned}
\mathscr{L} = \;& \bar{\psi}^{\alpha}_{A}\left(\partial\!\!\!/_{\mu} - (\mu + \delta\mu\sigma_3)\gamma^4 + m\right)^{\alpha\beta}_{AB}\psi^{\beta}_{B} + \tfrac{1}{2}\left(|\partial_\mu \Xi|\right)^2 + \tfrac{1}{2}\left(|\partial_\mu \Theta|\right)^2 \\
& + \frac{m_\Theta^2}{2}\Theta^{\alpha\beta}_{AB}\varepsilon_{ijkl}c^{i}_{\alpha A}c^{j}_{\beta B}c^{k}_{\gamma C}c^{l}_{\delta D}\Theta^{\gamma\delta}_{CD} \\
& + \frac{g^Y_\Theta}{2}\sqrt{\Xi^*}\;\varepsilon_{ijkl}c^{i}_{\alpha A}c^{j}_{\beta B}c^{k}_{\gamma C}c^{l}_{\delta D}\Theta^{\alpha\beta}_{AB}\psi^{\gamma}_{C}\psi^{\delta}_{D} \\
& + \frac{g^Y_\Theta}{2}\sqrt{\Xi}\;\varepsilon_{ijkl}c^{i}_{\alpha A}c^{j}_{\beta B}c^{k}_{\gamma C}c^{l}_{\delta D}\Theta^{\alpha\beta}_{AB}\bar{\psi}^{\gamma}_{C}\bar{\psi}^{\delta}_{D} \\
& + U(|\Xi|) + g^{\alpha\beta}_{AB}|\Xi|\Theta^{\alpha\gamma}_{AC}\Theta^{\beta\gamma}_{BC} + m_\Theta^2\;\Theta^{\alpha\beta}_{AB}\Theta^{\alpha\beta}_{AB}
\end{aligned}
$$

	ψ	$\bar{\psi}$	Ξ	Ξ^*	$\Theta^{\alpha\beta}_{AB}$
ψ	•	•			
$\bar{\psi}$	•	•			
Ξ			•	•	•
Ξ^*			•	•	•
$\Theta^{\alpha\beta}_{AB}$			•	•	•

Toy model: Bosonized Lagrangian $SU(2)_f \otimes SU(2)_c$

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$$+\frac{m^2_{\Theta}}{2}\Theta^{\alpha\beta}_{AB}\varepsilon_{ijkl}c^{i}_{\alpha A}c^{j}_{\beta B}c^{k}_{\gamma C}c^{l}_{\delta D}\Theta^{\gamma\delta}_{CD}$$

$$+\frac{g^{Y}_{\Theta}}{2}\sqrt{\Xi^*}\,\varepsilon_{ijkl}c^{i}_{\alpha A}c^{j}_{\beta B}c^{k}_{\gamma C}c^{l}_{\delta D}\Theta^{\alpha\beta}_{AB}\psi^{\gamma}_{C}\psi^{\delta}_{D}$$

$$+\frac{g^{Y}_{\Theta}}{2}\sqrt{\Xi}\,\varepsilon_{ijkl}c^{i}_{\alpha A}c^{j}_{\beta B}c^{k}_{\gamma C}c^{l}_{\delta D}\Theta^{\alpha\beta}_{AB}\bar{\psi}^{\gamma}_{C}\bar{\psi}^{\delta}_{D}$$

$$+U(|\Xi|) + g^{\alpha\beta}_{AB}|\Xi|\Theta^{\alpha\gamma}_{AC}\Theta^{\beta\gamma}_{BC} + m^2_{\Theta}\,\Theta^{\alpha\beta}_{AB}\Theta^{\alpha\beta}_{AB}$$

Flow equation for $U(\Xi)$

$$\partial_k U(\Xi) = \tfrac{1}{2}\mathrm{Tr}\left\{\left(\Gamma''_k + R_k\right)^{-1}\partial_k R_k\right\}$$

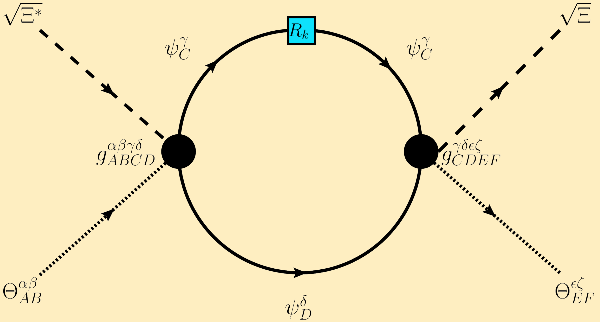
Coupling diagonal

$$(g_{\Xi\Theta})^{\alpha\beta\gamma\delta}_{ABCD} \sim \delta_{\alpha\gamma}\delta_{\beta\delta}\delta_{AC}\delta_{BD}$$

Three couplings

$$g^{(11)}_{\Xi\Theta},\ g^{(22)}_{\Xi\Theta},\ g^{(12)}_{\Xi\Theta} = g^{(21)}_{\Xi\Theta}$$

Fermionic contribution to coupling Θ-Θ-Ξ



The flow equation for the potential

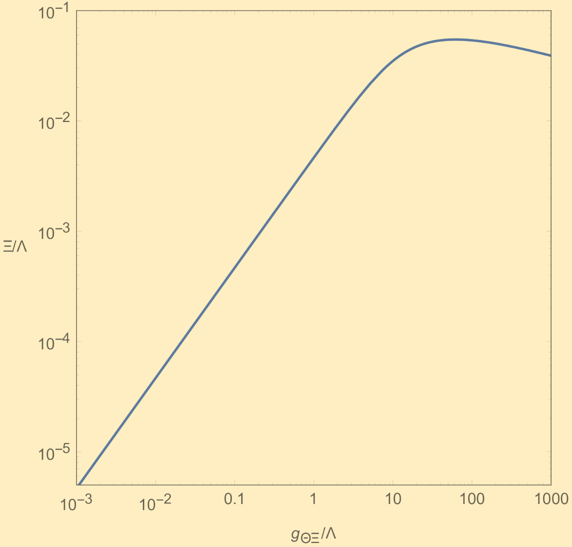
$$\frac{dU(|\Xi|)}{dk} = \frac{k^4}{6\pi^2}\left(\sum_{i\in\{11,22,12\}} \frac{2}{E_{\Theta}^{(i)}}\left(\frac{1}{2}+n_B\left(E_{\Theta}^{(i)}\right)\right)+\frac{1}{E_{\Xi}}\left(\frac{1}{2}+n_B(E_{\Xi})\right)\right)$$

$$E_{\Theta}^{(i)} \equiv \sqrt{k^2+m_{\Theta}^2+g_{\Xi\Theta}^{(i)}\,|\Xi|}$$

$$E_{\Xi} \equiv \sqrt{k^2+U_{\Xi\Xi}}$$

$$n_B(E) = \frac{1}{\exp\{\frac{E}{T}\}-1}$$

Condensate, dependent on $g_{\Xi\Theta}$, $\Lambda = 1\ GeV$, $\mu_1 = \mu_2$



Summary and Outlook

Summary

At strong coupling in fermionic systems with large asymmetry two fermion condensation is suppressed. Our toy model indicates, that in this scenario four fermion condensation can become a viable candidate.

Outlook I

- include flow of couplings $g_{\equiv\ominus}$

Outlook II

- study more realistic condensates


Thank You For Your Attention!